A Simple Parameterisation for Flux Footprint Predictions

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Wanted: Flux Footprint Estimates

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- Stochastic Lagrangian footprint models
- LES based footprint models
- Analytical footprint models

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- Stochastic Lagrangian footprint models $\rightarrow$ highly CPU-intensive
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Parameterisation for footprint estimates possible?

Scaling Parameters

z_m: Receptor height

h: Planetary boundary layer height

σ_w: Standard deviation of vertical velocity

u_*: Surface friction velocity

x: Alongwind distance from the receptor location

$\overline{f^y}$: Crosswind-integrated flux footprint function

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z_0: Roughness length

Flux Footprint Scaling

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\begin{aligned}
\Pi_1 &= z_m \overline{f^y} \\
\Pi_2 &= x/z_m \\
\Pi_3 &= h/(h - z_m) \\
\Pi_4 &= \sigma_w/u_*
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$$\begin{aligned}
X_* &= \Pi_4^{\alpha_1} \, \Pi_2 \\
F_* &= \Pi_4^{\alpha_2} \, \Pi_3 \, \Pi_1
\end{aligned}$$

X_*: Non-dimensional alongwind distance

F_*: Non-dimensional crosswind-integrated footprint function

α_1, α_2: Optimisation parameters

Flux Footprint Scaling

$$X_* = \left(\frac{\sigma_w}{u_*}\right)^{\alpha_1} \frac{x}{z_m}$$

$$F_* = \left(\frac{\sigma_w}{u_*}\right)^{\alpha_2} \left(1 - \frac{z_m}{h}\right)^{-1} z_m \overline{f^y}$$

σ_w: Standard deviation of vertical velocity

u_*: Surface friction velocity

z_m: Receptor height

α_1, α_2: Optimisation parameters

Testing of Scaling Procedure

- Ideally: test against observations
- Availability of applicable experimental data sets very limited
- Test against modelled data, footprint simulations

Footprint Model LPDM-B

- 3D Lagrangian stochastic footprint model (Kljun et al., 2002)
- Well-mixed for stable to convective stratification, as well as for receptors above the surface layer
- Backward time frame (Flesch et al., 1995)
- Evaluated using wind-tunnel data
- Dispersion module tested using water-tank and field data (Rotach et al., 1996; Rotach, 2001)

Range of Footprint Examples

Scenario	u_* [m s^{-1}]	w_* [m s^{-1}]	L [m]	h [m]
strongly convective	0.2	2.0	-5	2000
convective	0.2	1.0	-30	1500
slightly convective	0.3	0.5	-650	1200
neutral	0.5	0.0	∞	1000
slightly stable	0.4	–	1000	800
stable	0.3	–	130	250
strongly stable	0.3	–	84	200

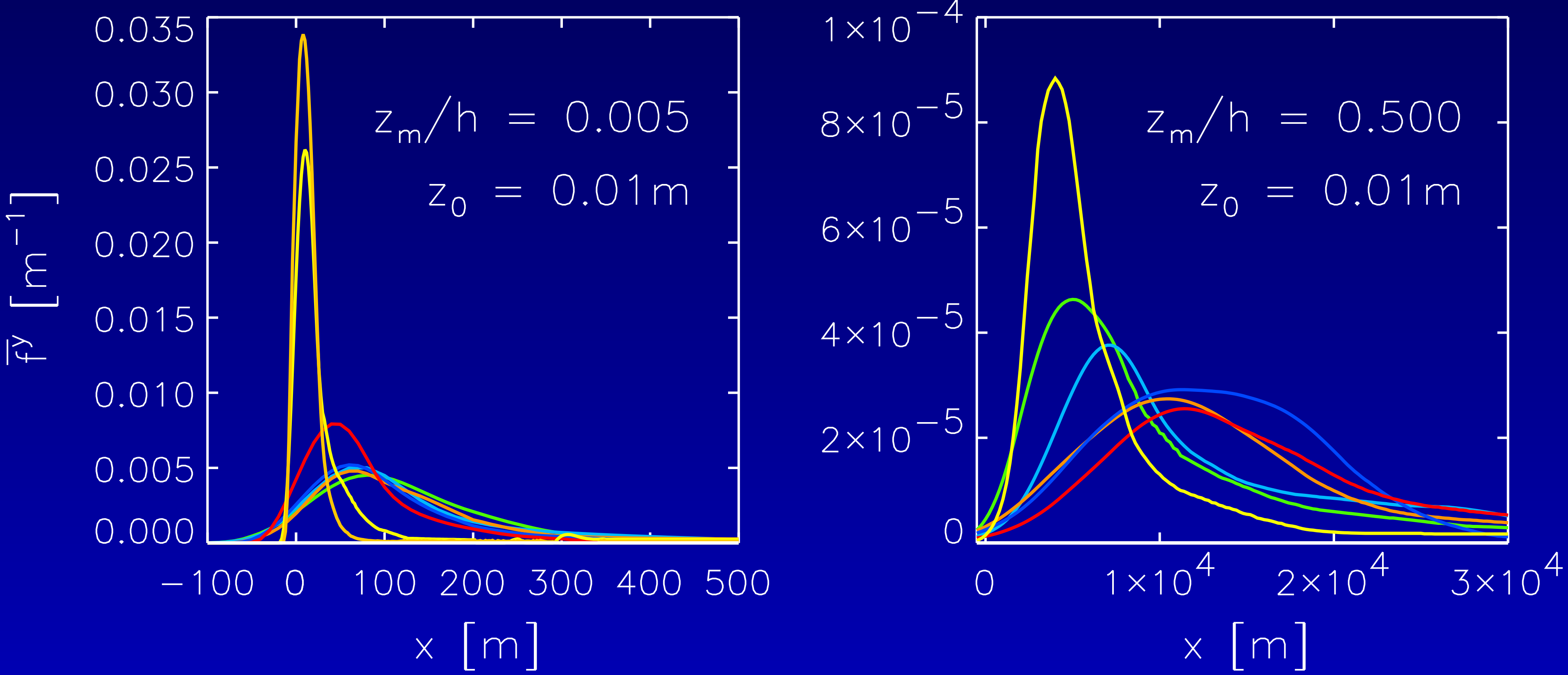
Range of Footprint Examples

Scenarios are run for

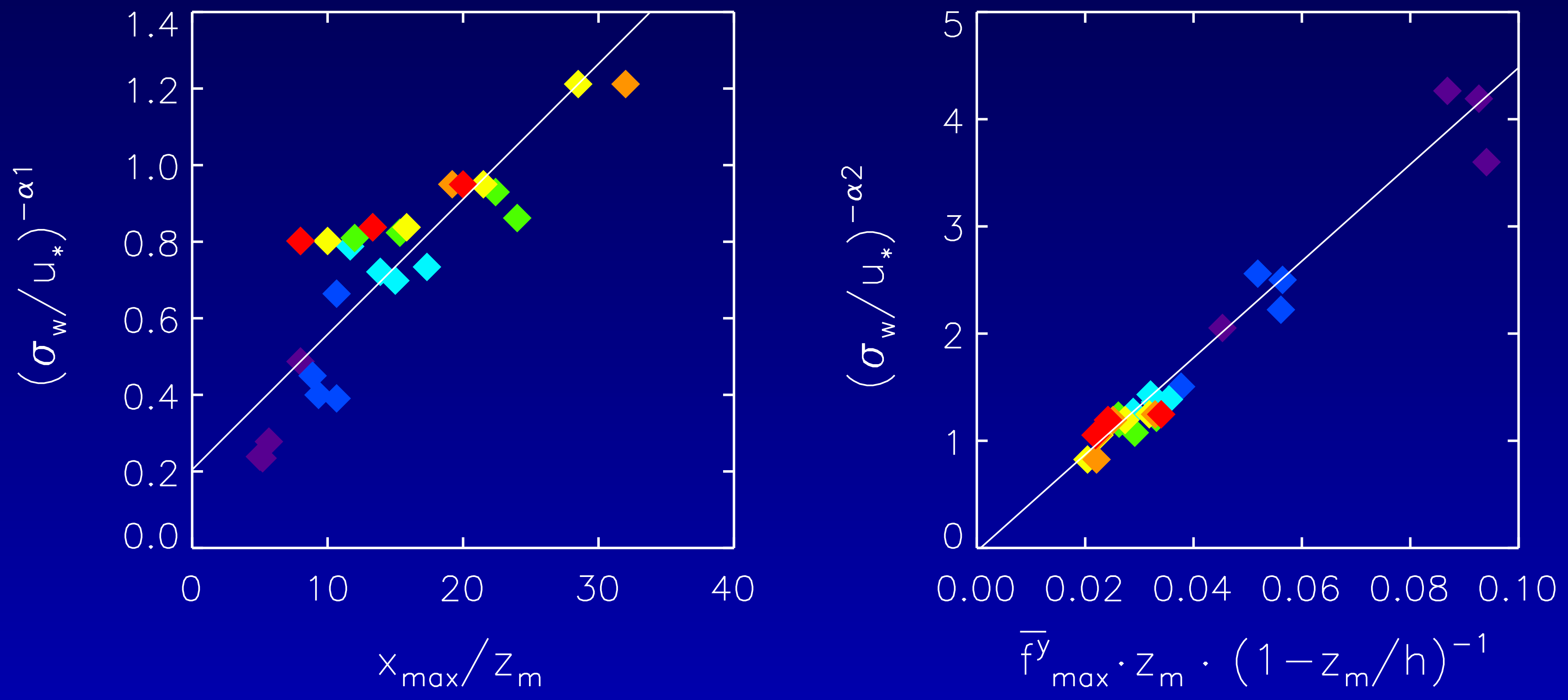
- Receptor heights, z_m / h:
  0.005, 0.01, 0.075, 0.25, 0.50

- Roughness length, z_0:
  0.01, 0.1, 0.3, 1.0 m

Crosswind-Integrated Flux Footprint



Determination of Optimisation Parameters



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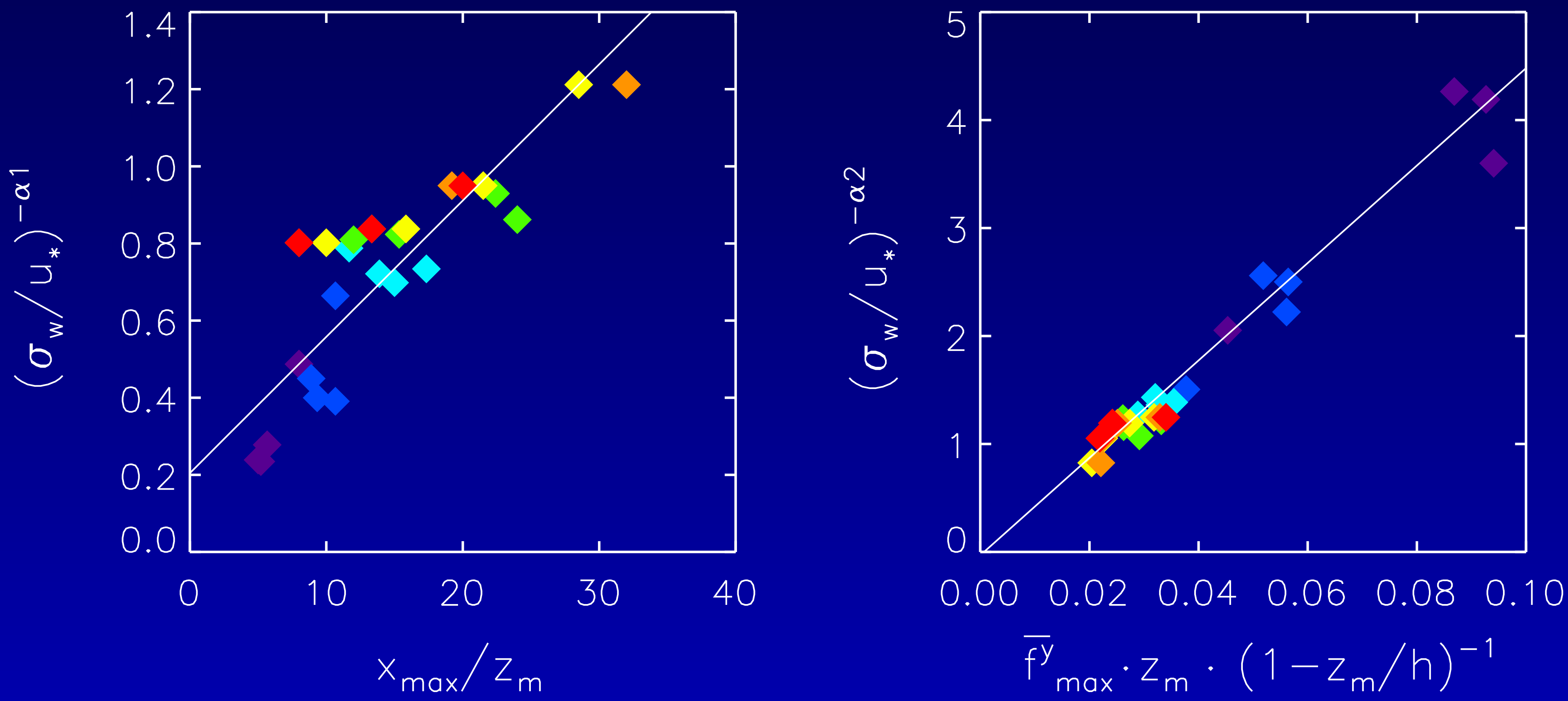
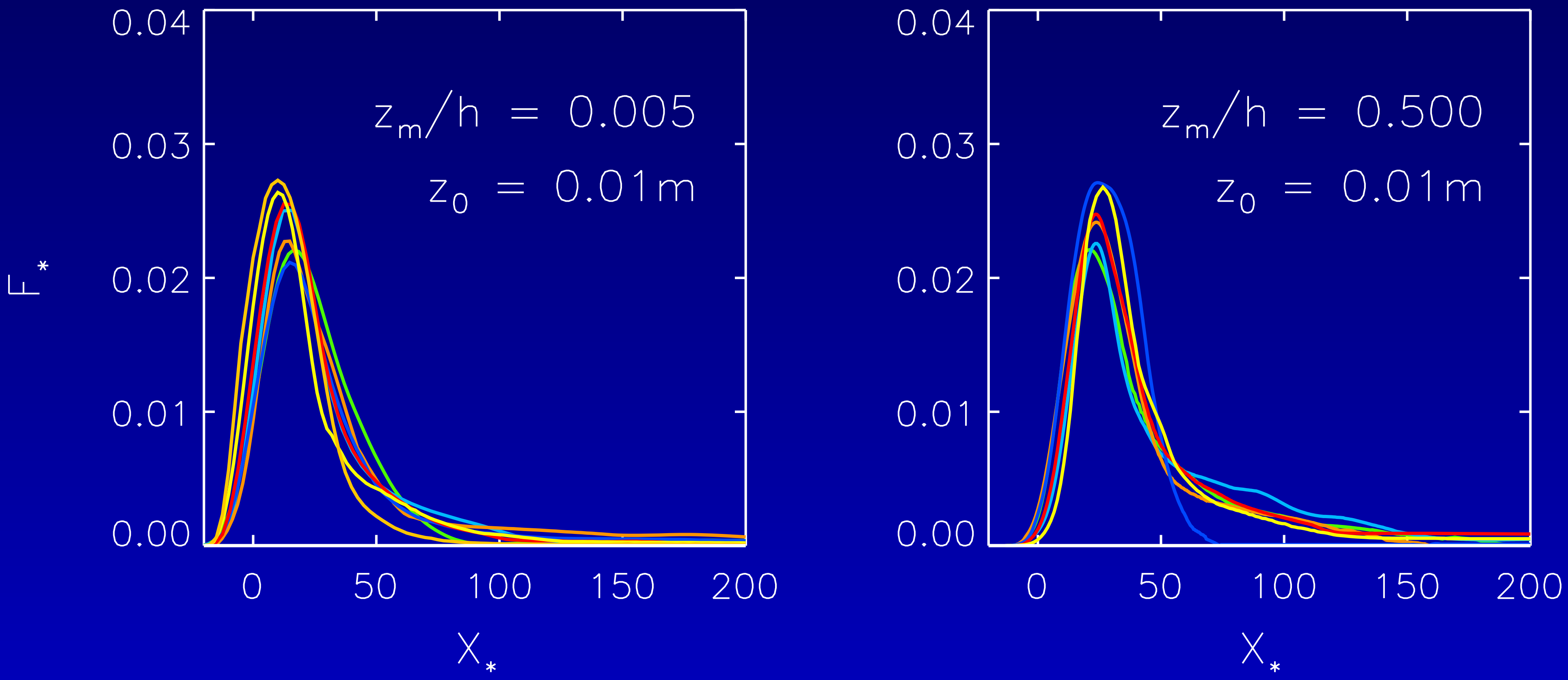


$$\rightarrow \alpha_1 = -\alpha_2 = 0.8$$

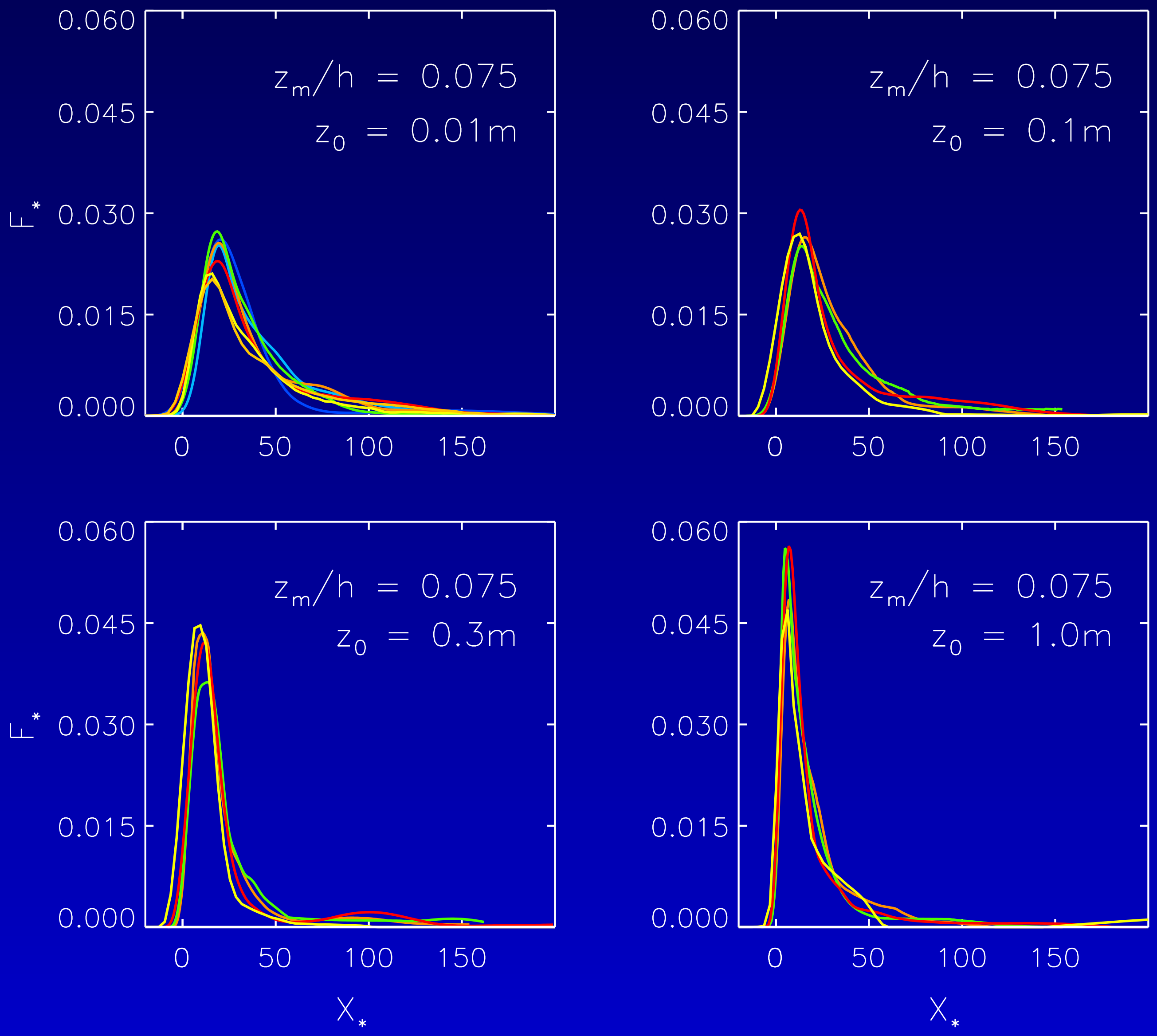
Flux Footprint after Scaling



Footprint Estimates for a Given Site

- Calculate a single footprint estimate for a given site using a suitable model
- Derive a scaled footprint estimate
- Use scaled footprint to estimate changes in real-scale footprint due to variations in stability

Influence of the Roughness Length

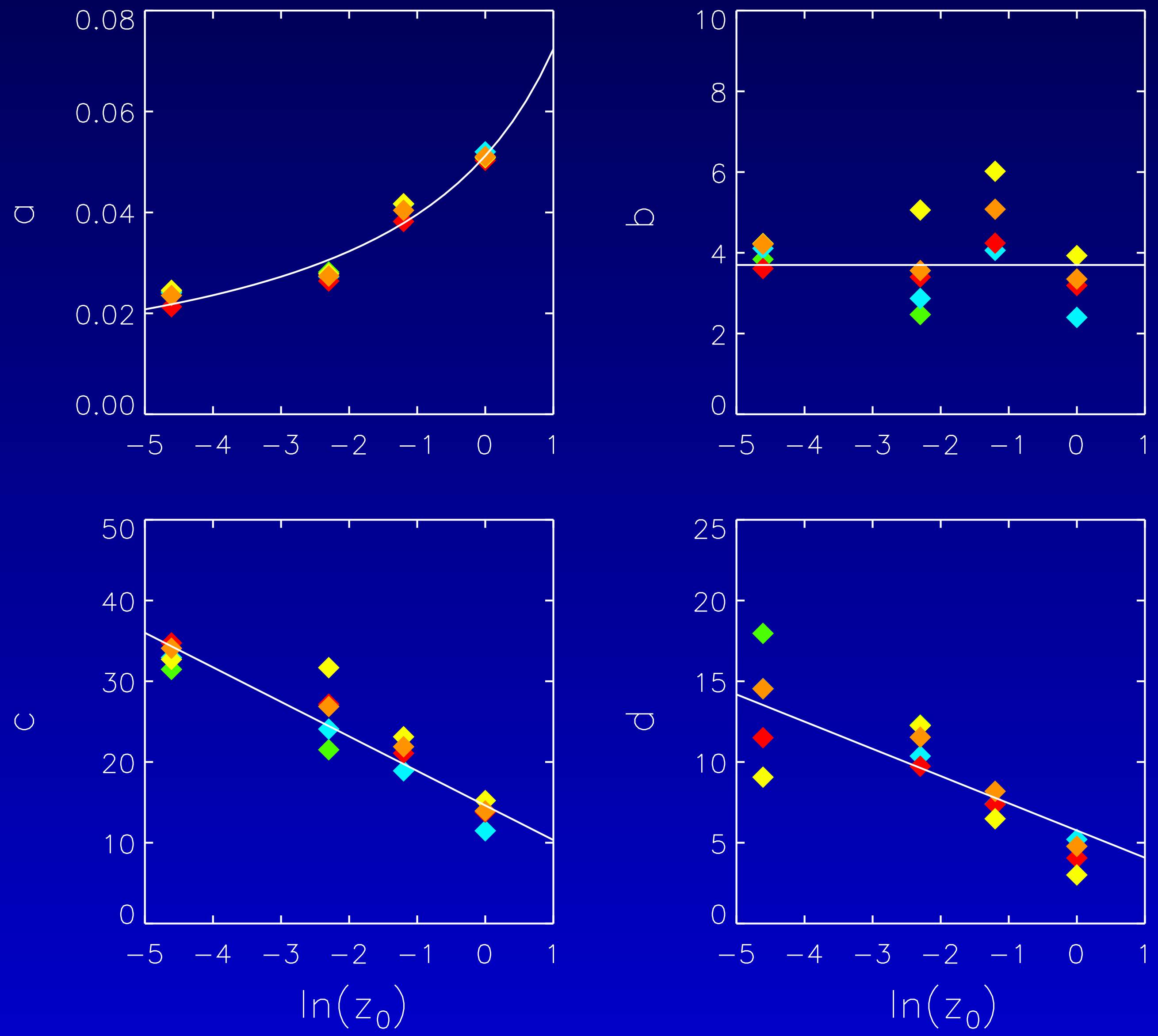


Flux Footprint Parameterisation

$$\hat{F}_* = a\left(\frac{\hat{X}_* + d}{c}\right)^b \exp\left\{b\left(1 - \frac{\hat{X}_* + d}{c}\right)\right\}$$

- $\hat{F}_*(\hat{X}_*)$ is the parameterisation of $F_*(X_*)$
- a, b, c, d: fitting parameters
- Use footprint model results to derive a, b, c, d

Fitting Parameters



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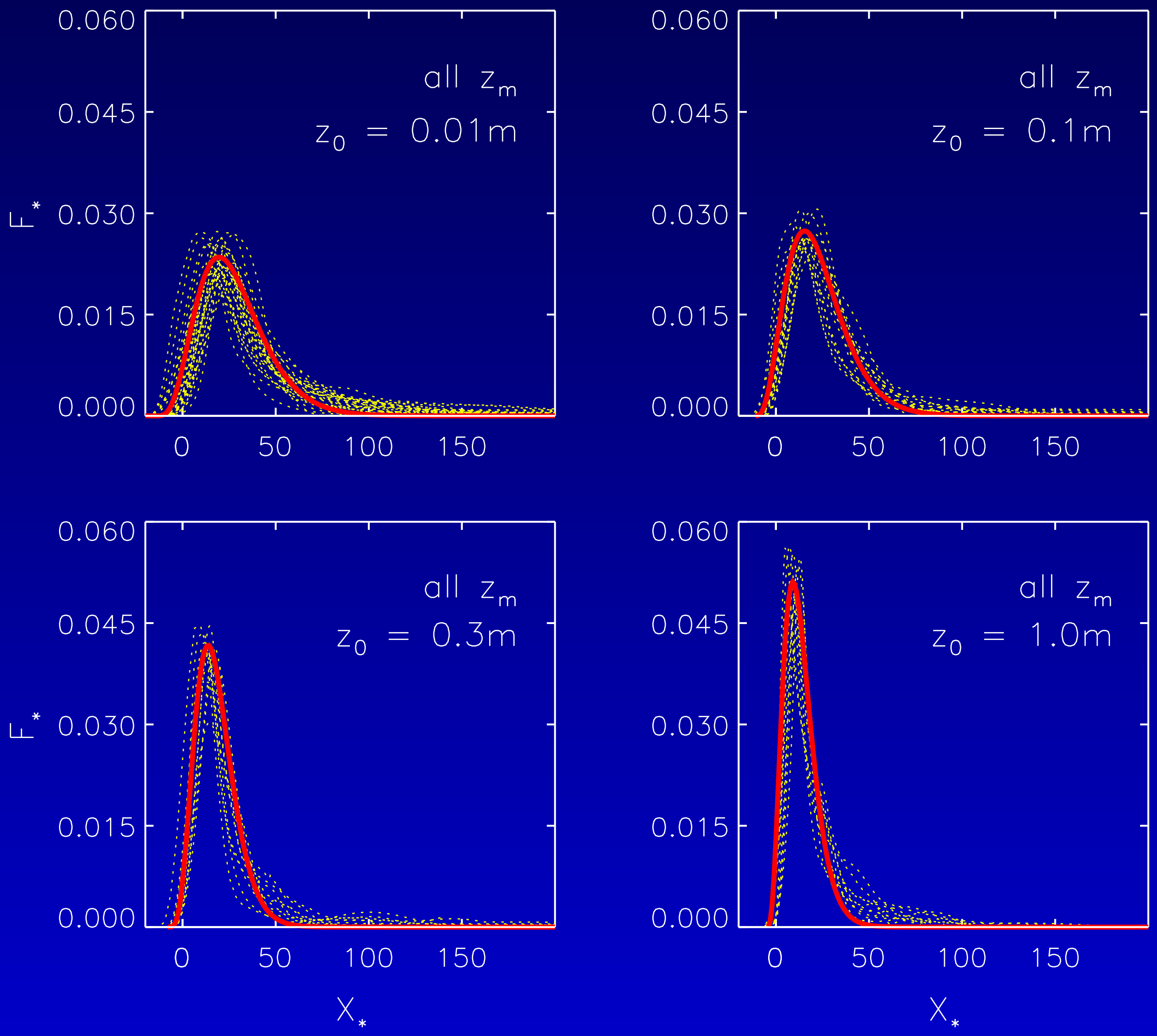

Applications

Peak location of footprint function:

Location of maximum influence for the measurement

$$
\begin{aligned}
x_{\max} &= X_{*,\max}\, z_m \left(\frac{\sigma_w}{u_*}\right)^{-\alpha_1} \\
&\approx \hat{X}_{*,\max}\, z_m \left(\frac{\sigma_w}{u_*}\right)^{-\alpha_1} \\
&\approx 2.59\,(3.42 - \ln z_0)\; z_m \left(\frac{\sigma_w}{u_*}\right)^{-0.8}
\end{aligned}
$$

Applications

Streamwise dimension of the footprint:

e.g., distance from receptor which includes 80% of the area influencing the measurement

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x_R &= X_{*,R}\; z_m \left(\frac{\sigma_w}{u_*}\right)^{-\alpha_1} \\
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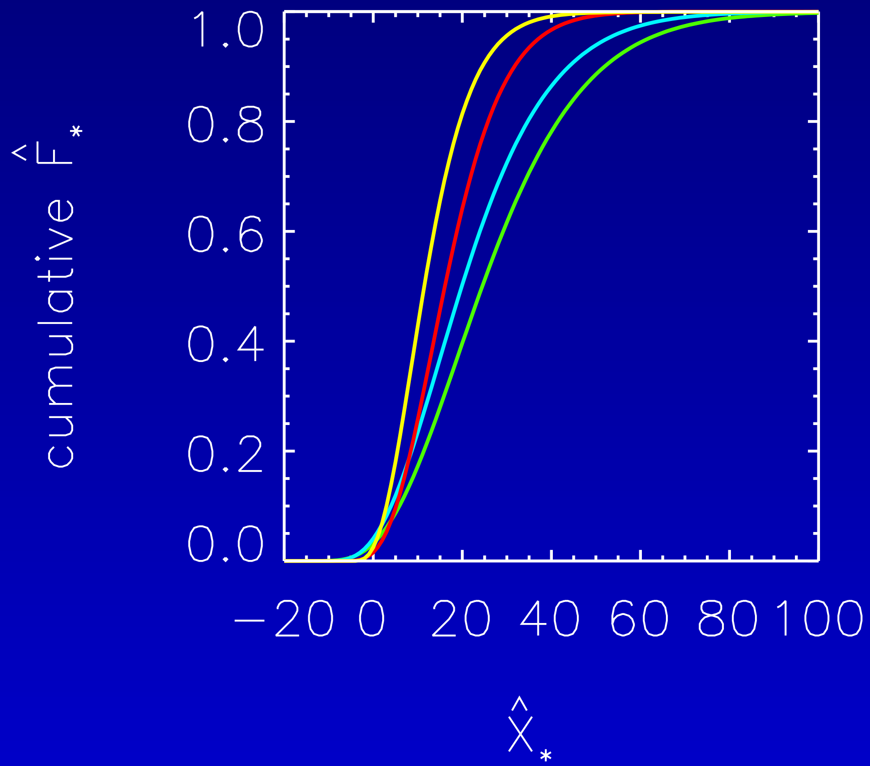

Summary

- Scaling approach for footprint functions based on key variables easily derived from common turbulence measurements

- Approach yields a single non-dimensional ‘master footprint function’ for a wide range of receptor heights and boundary-layer stabilities

- Parameterisation of scaled footprint ensemble allows for a quick assessment of the maximum influence location for the measurement and the streamwise dimension of the footprint

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http://footprint.kljun.net

Restrictions

$$\begin{aligned} -200 &\leq z_m/L \leq 1 \\ u_* &\geq 0.2\,\mathrm{m\,s^{-1}} \\ z_m &> 1\ \mathrm{m} \end{aligned}$$

Dynamically homogeneous terrain